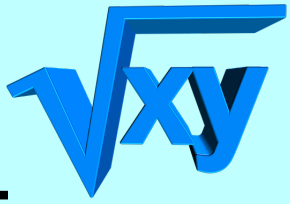
Math 115

Fall 2017

Lecture 25



Ch. 6

Rational Expression $= \dfrac{\text{Polynomial}}{\text{Polynomial}}$

$\dfrac{2x-3}{x+5}$, $\dfrac{x^2-5x}{x^2-7x+10}$, $\dfrac{x^2-3x+2}{x^2-7x-6}$

To Simplify

1) factor numerator completely

2) factor denominator completely

3) Cross out any common factors

Reduce $\dfrac{x^2-4x}{x^2-16} = \dfrac{x(x-4)}{(x+4)(x-4)}$

$= \boxed{\dfrac{x}{x+4}}$

Simplify $\dfrac{x^2+8x+15}{x^2-9} = \dfrac{(x+3)(x+5)}{(x+3)(x-3)}$

$= \boxed{\dfrac{x+5}{x-3}}$

Simplify: $\dfrac{2x^2+3x-5}{x^2+4x-5} = \dfrac{(x-1)(2x+5)}{(x+5)(x-1)}$

$= \boxed{\dfrac{2x+5}{x+5}}$

$2x^2+3x-5$

$P=-10$

$S=3$

-10

$-2 \& 5$

$2x^2-2x+5x-5$

$2x(x-1)+5(x-1)$

Excluded Values: Are those values that make the denominator equal to zero.

How to find them:

1) Deno = 0

2) Solve

Find all excluded values:

$$\frac{x-1}{2x+5}$$

$2x+5=0$

$2x=-5$

$x=-5/2$

E.V. $-\frac{5}{2}$

Find all excluded values:

$$\frac{x-8}{x^2+5x-6}$$

Solve → Deno. = 0, then Solve

$x^2+5x-6=0$

$(x+6)(x-1)=0$

by Z.F.P.

$x+6=0$ $\quad$ $x-1=0$

$x=-6$ $\quad$ $x=1$

E.V.: -6 & 1

Find all excluded values:

$$\frac{x}{x^2-100}$$

↳ Deno. = 0, Solve

$x^2 - 100 = 0$

$(x+10)(x-10) = 0$

by Z.F.P.

$x+10=0$ or $x-10=0$

$x=-10$ $\quad x=10$

E.V. are ± 10

Basic math: Simplify $\frac{15}{35} \cdot \frac{14}{18}$

$$= \frac{\cancel{5}\cdot 3}{\cancel{5}\cdot 7} \cdot \frac{\overset{1}{\cancel{7}}\cdot \cancel{2}}{3\cdot 3\cdot \cancel{2}}$$

$$= \boxed{\frac{1}{3}}$$

Multiply

$$\frac{2x+8}{x^2+4x} \cdot \frac{3x-5}{4x-20}$$

$$= \frac{\cancel{2}(x+4)}{x(x+4)} \cdot \frac{1(3x-5)}{\underset{2}{\cancel{4}}(x-5)} = \boxed{\frac{3x-5}{2x(x-5)}}$$

Multiply: $\dfrac{x^2-25}{x^2+7x+10} \cdot \dfrac{x^2-9}{x^2-8x+15}$

$= \dfrac{(x+5)(x-5)}{(x+2)(x+5)} \cdot \dfrac{(x+3)(x-3)}{(x-3)(x-5)}$

$= \boxed{\dfrac{x+3}{x+2}}$

Multiply

$\dfrac{x^2-4}{2x^2-3x-5} \cdot \dfrac{x^2+6x+5}{x^2-4x+4}$

$= \dfrac{(x-2)(x+2)}{(x+1)(2x-5)} \cdot \dfrac{(x+5)(x+1)}{(x-2)(x-2)}$

$= \boxed{\dfrac{(x+2)(x+5)}{(2x-5)(x-2)}}$

$2x^2-3x-5 = 2x^2+2x-5x-5$

$= 2x(x+1)-5(x+1)$

$P=-10$, $S=-3$, -10

$2 \quad -5$

Basic Math $\frac{24}{45} \div \frac{16}{18} = \frac{24}{45} \cdot \frac{18}{16}$

Divide

$= \frac{8 \cdot 3}{9 \cdot 5} \cdot \frac{9 \cdot 2}{8 \cdot 2}$

$= \boxed{\frac{3}{5}}$

$\frac{x-4}{2x+3} \div \frac{x^2+2x-24}{2x^2+3x}$

$= \frac{x-4}{2x+3} \cdot \frac{2x^2+3x}{x^2+2x-24} = \frac{x-4}{2x+3} \cdot \frac{x(2x+3)}{(x-4)(x+6)}$

$= \boxed{\frac{x}{x+6}}$

Divide: $\frac{2x^2-11x+5}{5x-25} \div \frac{4x-2}{10}$

$= \frac{(2x-1)(x-5)}{5(x-5)} \cdot \frac{10^{2}}{2(2x-1)}$

$= \frac{2}{2} = \boxed{1}$

Divide: $\dfrac{7x^2-35x}{x^2-25} \div \dfrac{x^2+4x}{x^2+3x-10}$

$= \dfrac{7x(x-5)}{(x+5)(x-5)} \cdot \dfrac{(x+5)(x-2)}{x(x+4)}$

$= \boxed{\dfrac{7(x-2)}{x+4}}$

Simplify: $\dfrac{5x-20}{3x^2+x} \cdot \dfrac{3x^2+13x+4}{x^2-16}$

$= \dfrac{5(x-4)}{x(3x+1)} \cdot \dfrac{(3x+1)(x+4)}{(x+4)(x-4)}$

$= \boxed{\dfrac{5}{x}}$

Solve by using Z.F.P.:

$$(x-7)(2x+9)(3x-5)=0$$

RHS must be zero.
LHS must be factored.

$x-7=0$ $\quad$ $2x+9=0$ $\quad$ $3x-5=0$

$x=7$ $\quad$ $x=\frac{-9}{2}$ $\quad$ $x=5/3$

$$\left\{-\frac{9}{2}, 5/3, 7\right\}$$

Solve by factoring

$$2x^2=7x+9$$

$2x^2-7x-9=0$

$-9 \& 2$ $\quad$ $P=-18$ $\quad$ $S=-7$

18

$2x^2-9x+2x-9=0$

$x(2x-9)+1(2x-9)=0$

$\{-1, \frac{9}{2}\}$

① RHS = 0

② LHS must be factored

③ use Z.F.P.

$(2x-9)(x+1)=0$

use Z.F.F.

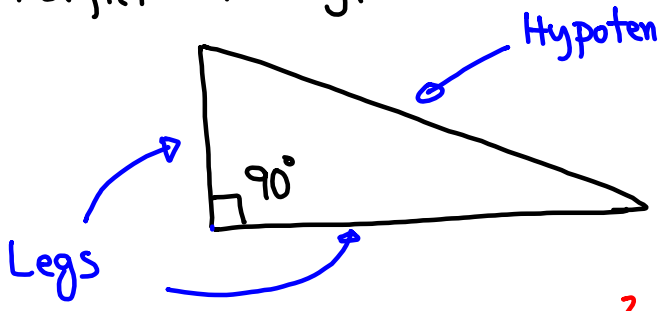
$2x-9=0$

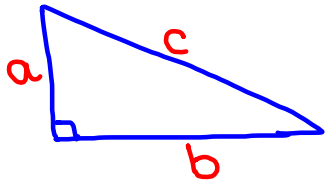
$x=\frac{9}{2}$

$x+1=0$

$x=-1$

Right Triangle

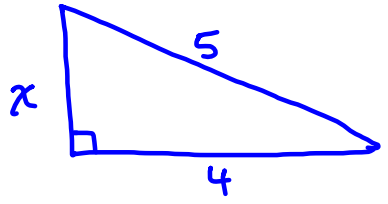




$a^2 + b^2 = c^2$

Pythagorean thm

find x:



by Pythagorean thm

$x^2 + 4^2 = 5^2$

$x^2 + 16 = 25$

$x^2 = 25 - 16$

$x^2 = 9$

$x = 3$ $\quad$ ~~$x = -3$~~

Also $x^2 + 16 = 25$

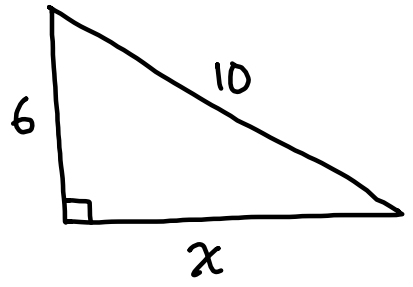
$x^2 + 16 - 25 = 0$

$x^2 - 9 = 0$

$(x+3)(x-3) = 0$

~~$x = -3$~~ $\quad$ $x = 3$

Find x:



Right Triangle

Use Pythagorean Thrm

$a^2 + b^2 = c^2$

$6^2 + x^2 = 10^2$

$36 + x^2 = 100$

$x^2 = 100 - 36$

$x^2 = 64$

$x = 8$ ~~$x = -8$~~

Also

$36 + x^2 = 100$

$36 + x^2 - 100 = 0$

$x^2 - 64 = 0$

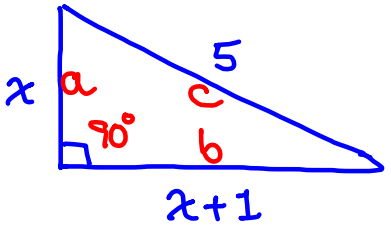
$(x+8)(x-8) = 0$

~~$x = -8$~~ $x = 8$

Find x

Right Triangle

Pythagorean thrm



$a^2 + b^2 = c^2$

$x^2 + (x+1)^2 = 5^2$

$x^2 + (x+1)(x+1) = 25$

$x^2 + x^2 + x + x + 1 = 25$

$2x^2 + 2x + 1 - 25 = 0$

$2x^2 + 2x - 24 = 0$

Divide by 2 to reduce

$x^2 + x - 12 = 0$

$(x+4)(x-3) = 0$

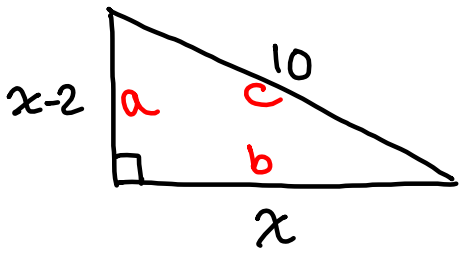
~~$x = -4$~~ $x = 3$

find x:

Right Triangle

Pythagorean Thm

$a^2 + b^2 = c^2$



$$(x-2)^2 + x^2 = 10^2$$

$$(x-2)(x-2) + x^2 = 100$$

$$x^2 - 2x - 2x + 4 + x^2 = 100$$

$$2x^2 - 4x + 4 - 100 = 0$$

$$2x^2 - 4x - 96 = 0$$

$2x^2 - 4x - 96 = 0$

Divide by 2 or factor out 2.

$$x^2 - 2x - 48 = 0$$

$$(x-8)(x+6) = 0$$

$x = 8$

~~$x = -6$~~

Solve $x^2 - 6x + 9 = 0$ by Quadratic Formula.

$ax^2 + bx + c = 0$

$b^2 - 4ac = (-6)^2 - 4(1)(9) = 36 - 36 = \boxed{0}$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-(-6) \pm \sqrt{0}}{2(1)}$$

$$= \frac{6 \pm 0}{2} = \frac{6}{2} = \boxed{3}$$

$\{3\}$

Solve $4x^2 + 20x + 25 = 0$ by Quadratic Formula.

$a = 4$, $b = 20$ $c = 25$

$b^2 - 4ac = (20)^2 - 4(4)(25) = 0$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-20 \pm \sqrt{0}}{2(4)} = \frac{-20}{8_2} = \boxed{-\frac{5}{2}}$$

$\{-5/2\}$

Consider $(2x+3)(x+4)=25$

① Foil, Simplify, and write in $ax^2+bx+c=0$ form

$$2x^2+8x+3x+12-25=0$$

$$2x^2+11x-13=0$$

② Identify a, b, and c, then compute

$$b^2-4ac=(11)^2-4(2)(-13)=\boxed{225}$$

③ use Quadratic formula to Solve

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-11\pm\sqrt{225}}{2(2)}$$

$$=\frac{-11\pm15}{4}$$

$x=\frac{-11+15}{4}=\frac{4}{4}=\boxed{1}$

$x=\frac{-11-15}{4}=\frac{-26}{4}=\boxed{\frac{-13}{2}}$